

The Role of Duty Cycle in Flicker-Induced Visual Hallucinations

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Introduction

Flickering lights produce visual hallucinations in the form of simple geometric patterns in the visual cortex. In a subjective study on perceived hallucinations from flickering light, spirals, targets and honeycombs were reported.¹ A study using firing rate models showed that cortical lateral inhibition produces instabilities at specific frequencies which allows for pattern formation. Flickering with higher frequencies that allow for period doubling produce stripes. Period doubling occurs because the network cannot keep up with higher frequency flickering so a period is twice the flickering period. Lower frequencies produce hexagons.²

In this study, the effects of the flicker's duty cycle, the proportion of the flicker's period which is in an active state, on the pattern formation are investigated.

Model

The primary visual cortex is modeled with a spatially distributed excitatory inhibitory layered Wilson-Cowan firing rate model. $\tau_{e,i}$ is the time constant of the layer, $U_{e,i}(x)$ is the population activity, $F(u) = 1/(1 + \exp(-u))$ is a nonlinearity representing firing rate, a_{xy} is x->y layer connection strength, $\theta_{e,i}$ is the threshold, and $g_{e,i}$ is the stimulus strength.

$$\begin{aligned}\tau_e \frac{dU_e}{dt} &= -U_e(x, t) + F(a_{ee}K_e(x) * U_e(x, t) - a_{ie}K_i(x) * U_i(x, t) - \theta_e + g_e S(t)) \\ \tau_i \frac{dU_i}{dt} &= -U_i(x, t) + F(a_{ei}K_e(x) * U_e(x, t) - a_{ii}K_i(x) * U_i(x, t) - \theta_i + g_i S(t))\end{aligned}$$

The spatially convolving kernel, $K_{e,i}(x)$, describes how the excitatory and inhibitory neural populations are locally connected. The neural populations are laid out in a square.

$$K_{e,i}(x) = \frac{1}{\pi \sigma_{e,i}^2} e^{-(x^2+y^2)/\sigma_{e,i}^2}$$

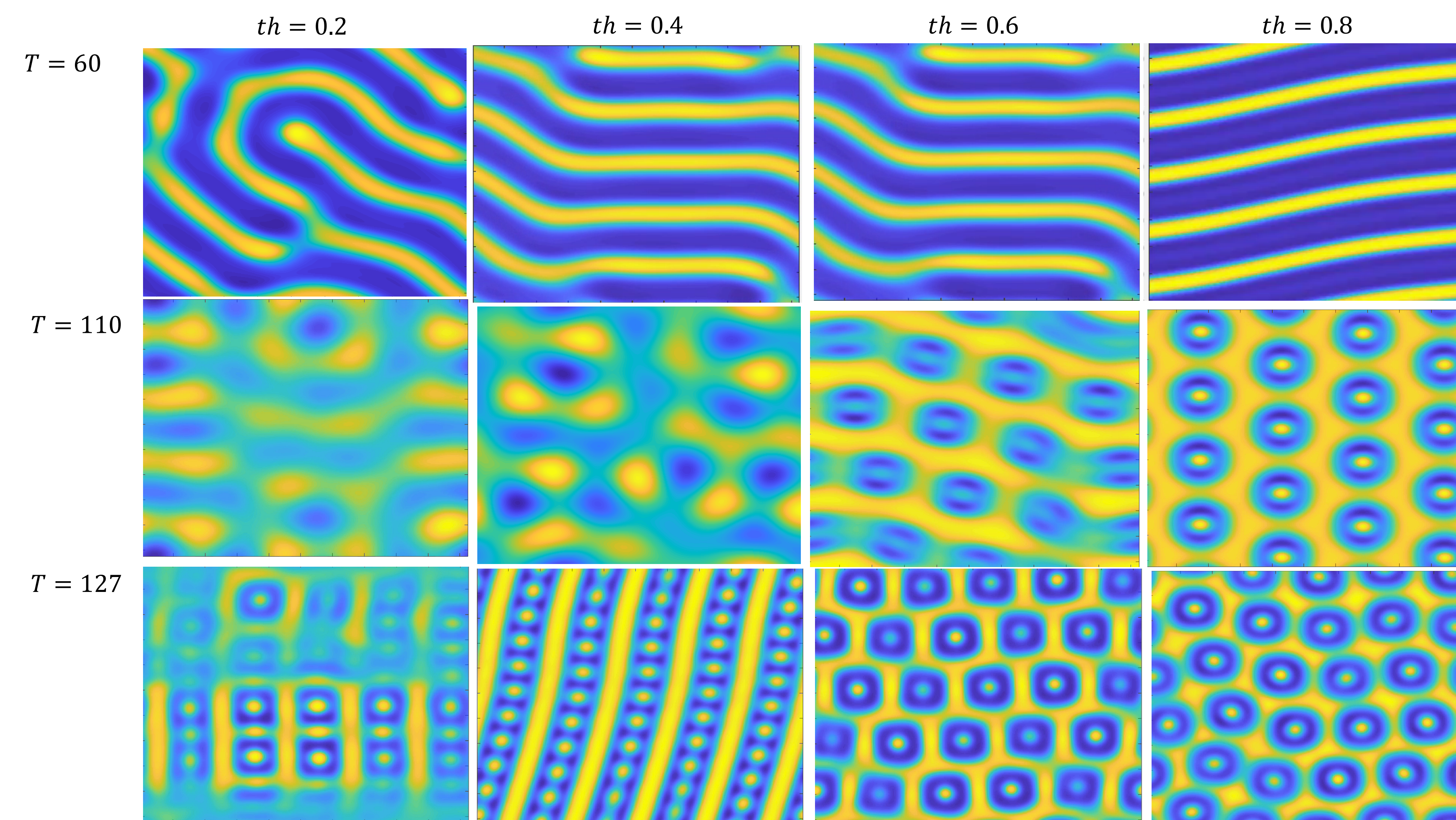
The stimulus is a square wave with the duty cycle corresponding to th . The stimulus is smoothed with the nonlinearity, H .

$$S(t) = H(\sin\left(\frac{2\pi t}{T}\right) - th)$$

The visual field is projected onto the cortex with radial distance from visual field center, ϵ , and azimuthal angle around the visual field, a .³

$$X = \ln(1 + \epsilon) \quad Y = -\frac{\epsilon a}{1 + \epsilon}$$

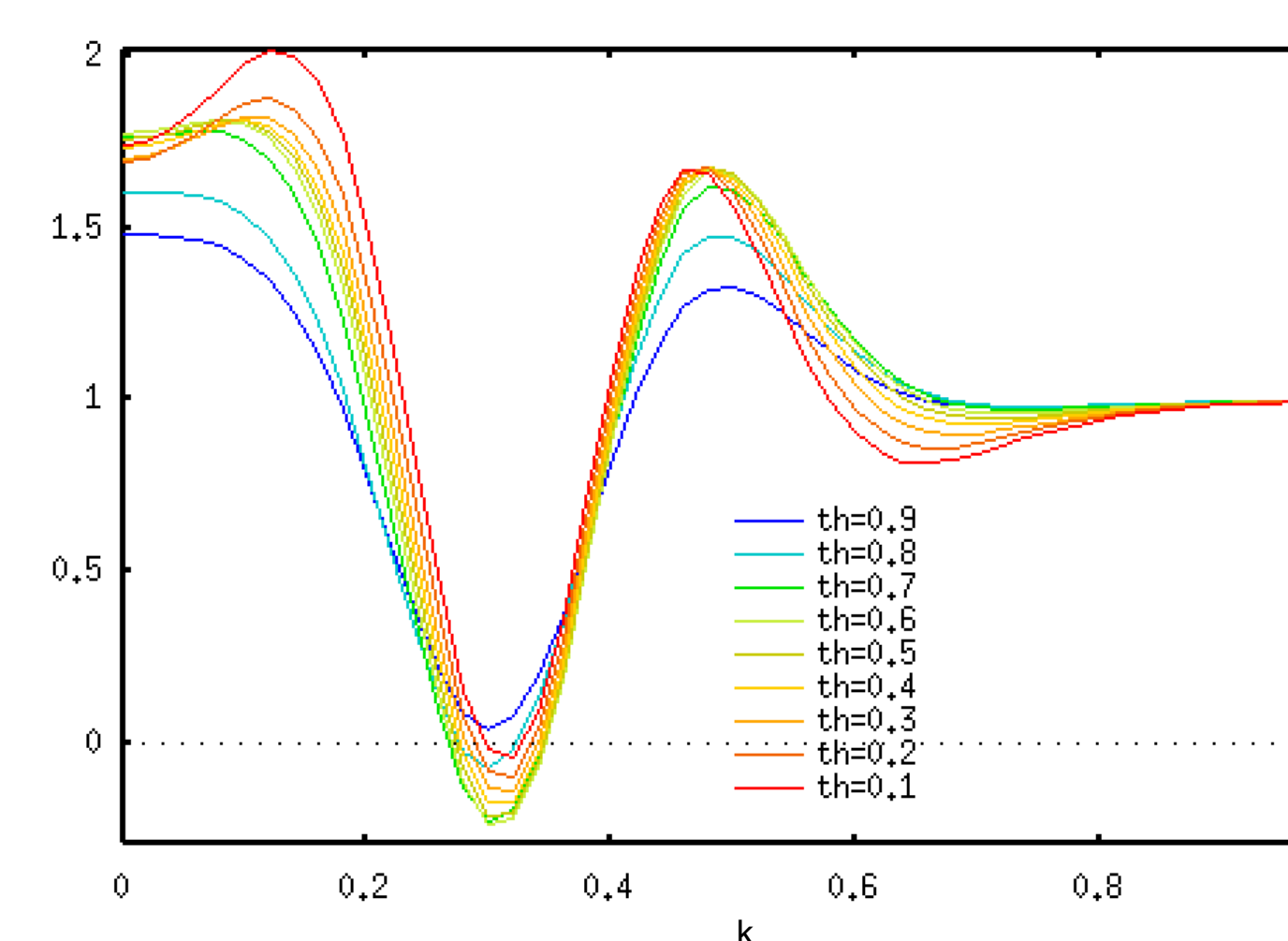
Pattern Formation Across Duty Cycle and Stimulus Period



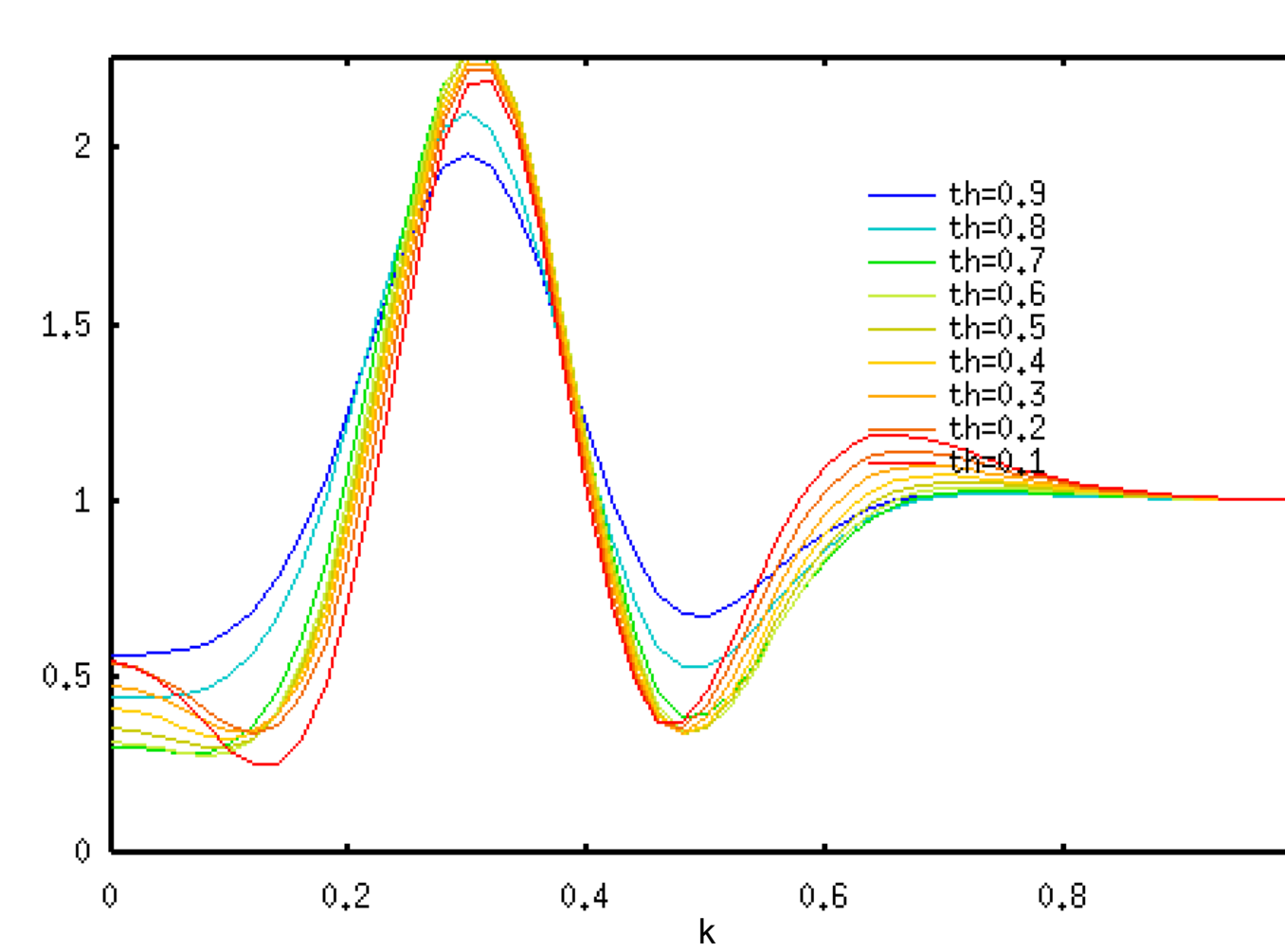
Parameters: $g_e = 0.8, g_i = 0, \tau_e = 10ms, \tau_i = 20ms, a_{EE} = 10, a_{EI} = 12, a_{IE} = 8.5, a_{II} = 10, \theta_e = 2, \theta_i = 3.5, \sigma_e = 10, \sigma_i = 25$

Stability Analysis

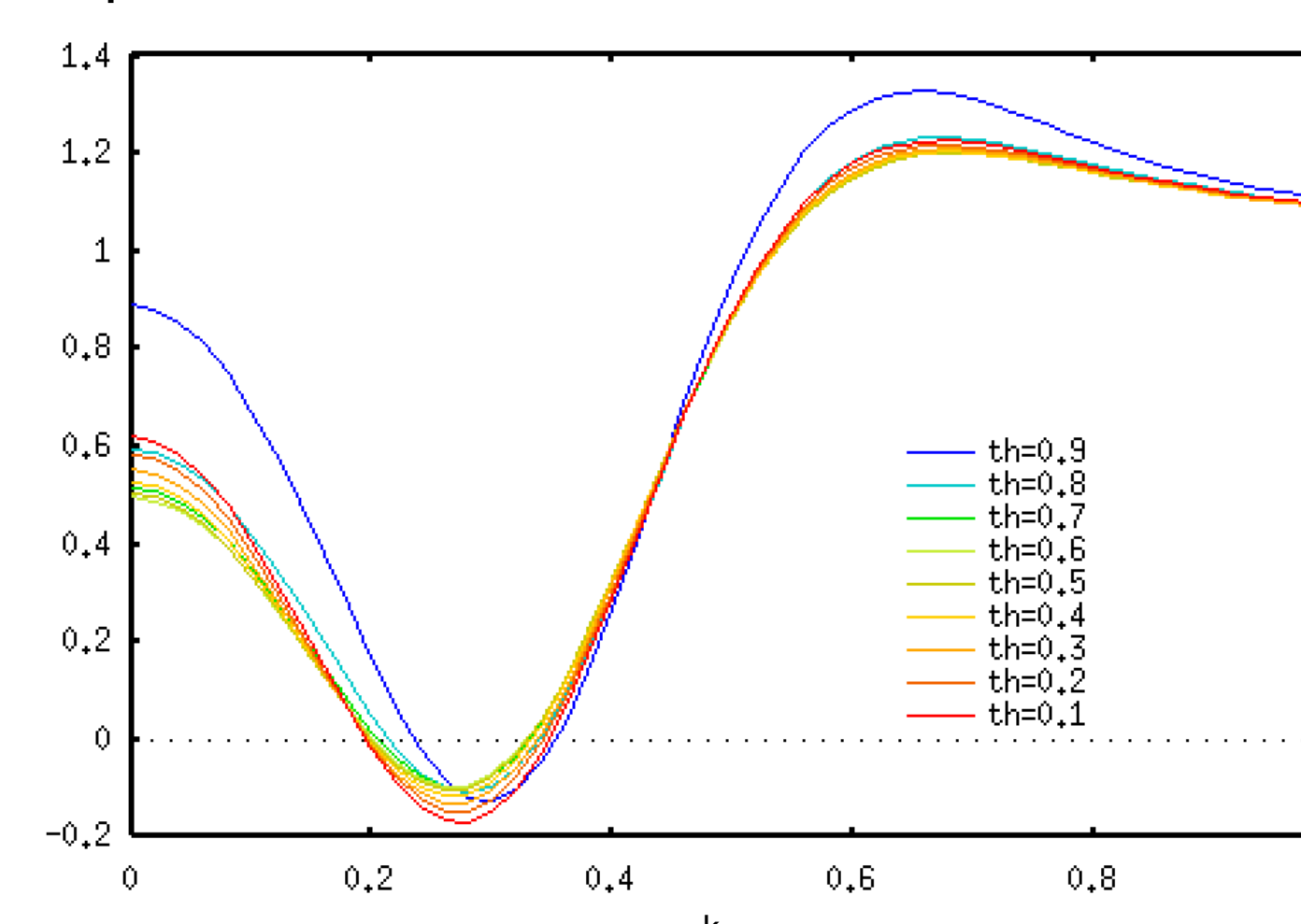
Stability conditions for +1 Floquet multiplier with respect to wave number, k , for non-period doubling solutions ($T=128$). Wave numbers below 0 correspond to instabilities that can produce patterns.



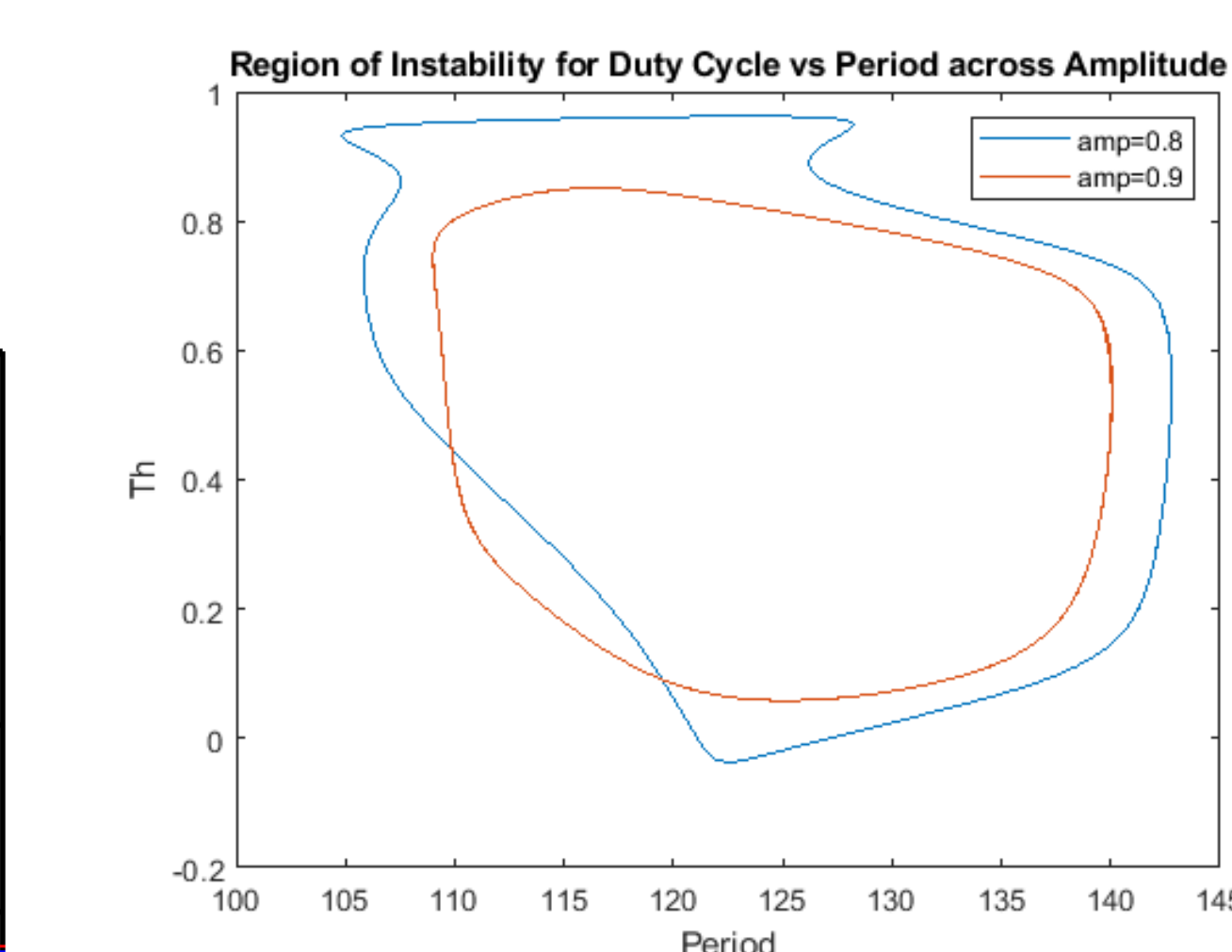
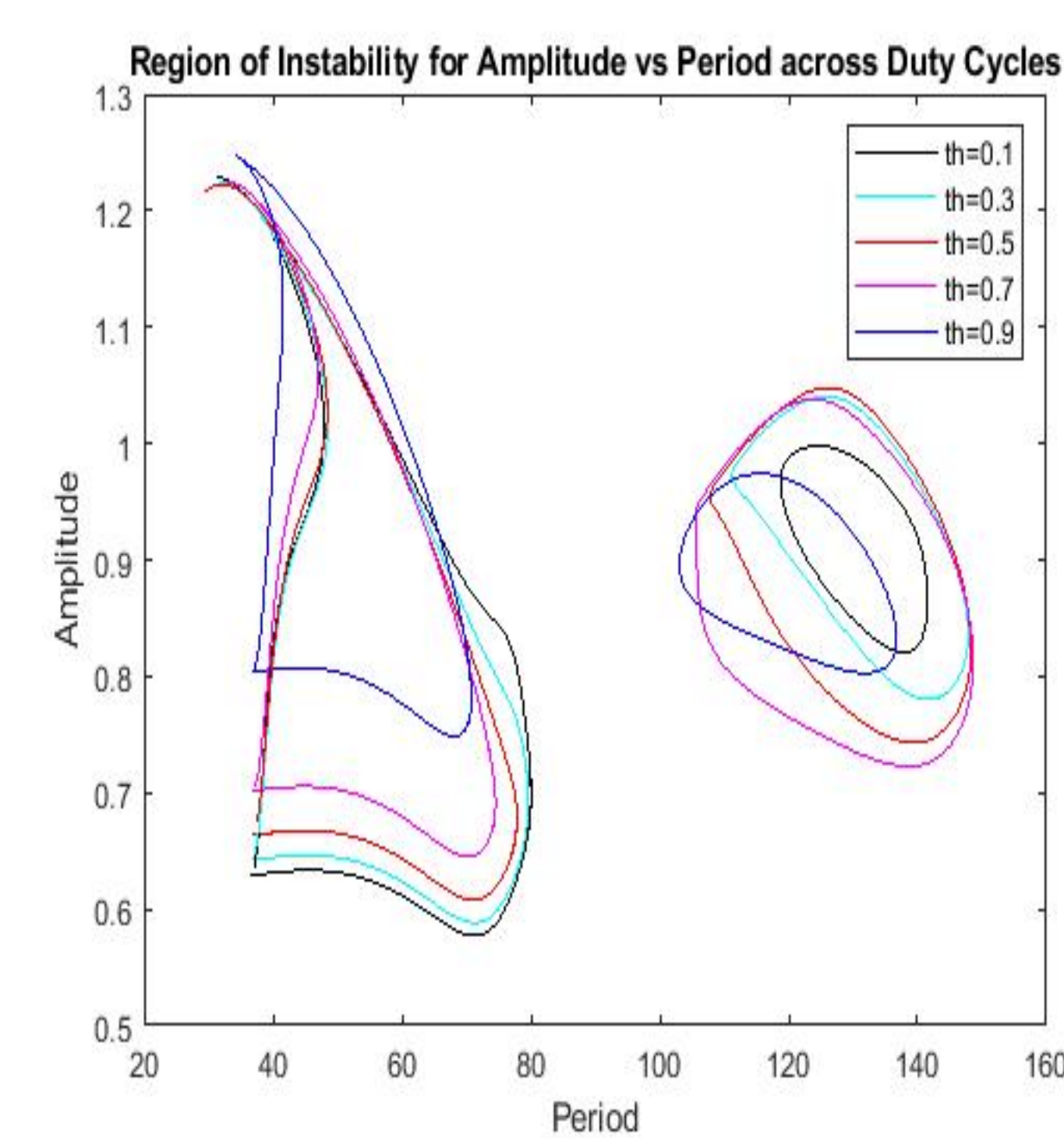
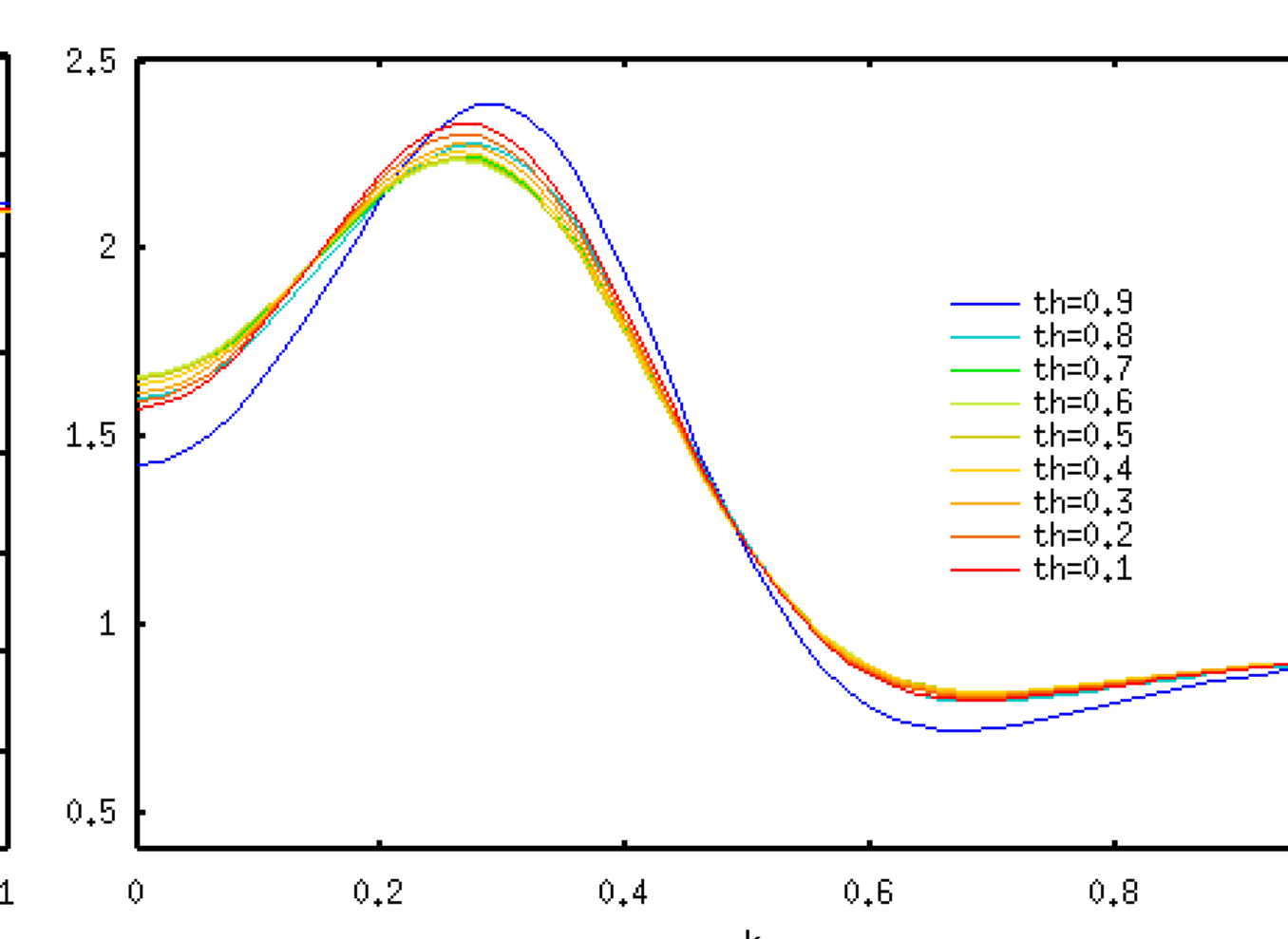
Stability conditions for -1 Floquet multiplier with respect to wave number, k , for non-period doubling solutions ($T=128$).



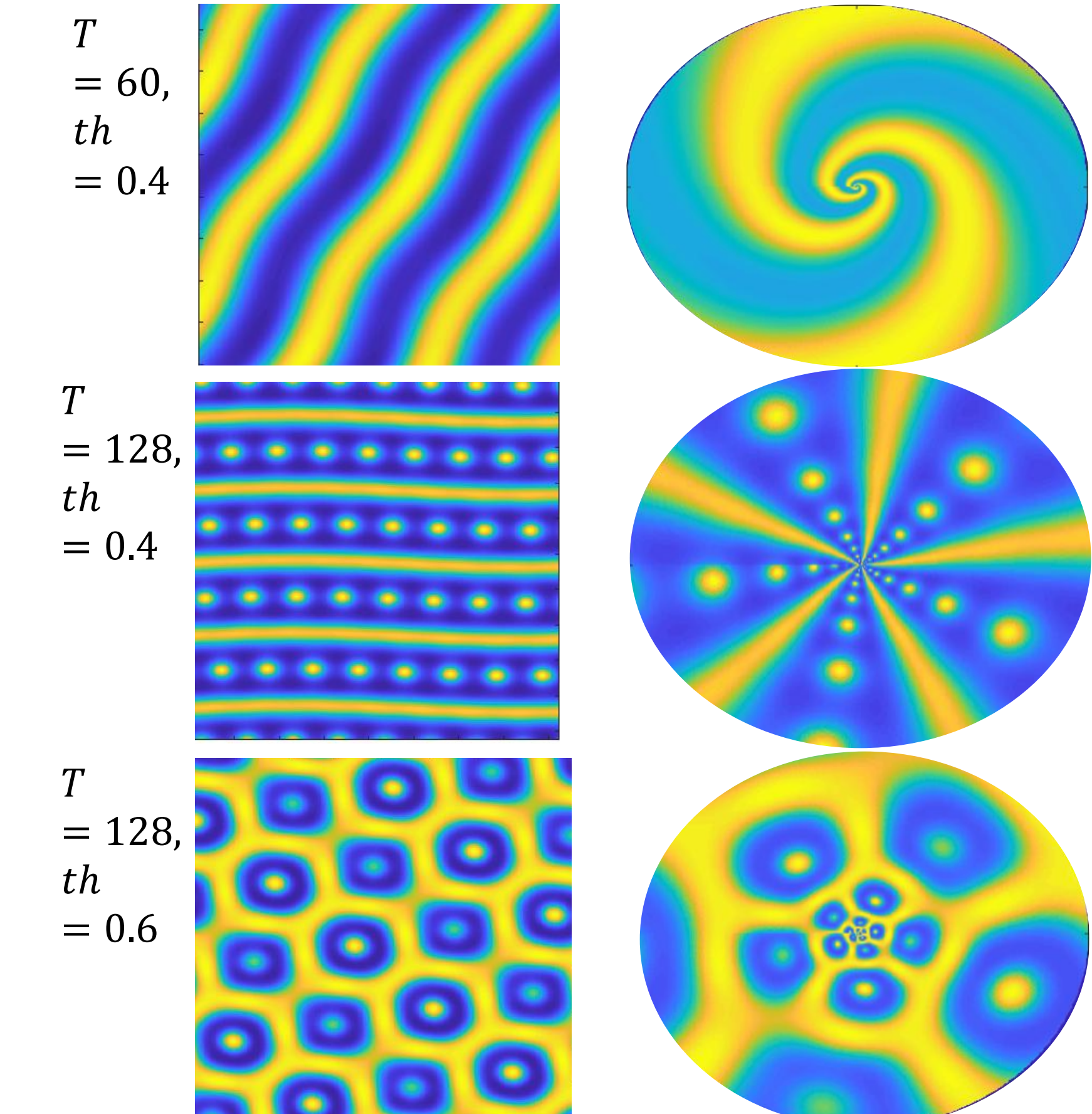
Stability conditions for -1 Floquet multiplier with respect to wave number, k , for period doubling solutions ($T=60$). Wave numbers below 0 correspond to instabilities that can produce patterns.



Stability conditions for +1 Floquet multiplier with respect to wave number, k , for period doubling solutions ($T=60$).



Visual Cortex to Retina Mapping



Discussion

Duty cycle plays a significant role in pattern formation for non period doubling frequencies ($T=110,127$). However, duty cycle does not have a significant effect in pattern formation for period doubling frequencies ($T=60$).

For the graph of +1 Floquet multiplier vs wave number, two local minima begin to emerge for higher duty cycle (lower th). The second local minimum has twice the wave number as the first. This creates a resonance of unstable Turing modes for the original spatial frequency, k , and double the wave number, $2k$. We hypothesize that the resonance in the two wave numbers allow for patterns such as the stripes with spots seen in $T=127$ and $th=0.4$. The stripes would correspond to the first local minimum and the spots would correspond to the second local minimum.

In the period doubling cases, the Floquet multiplier curves do not change significantly for different duty cycles. These findings support the stimulations which show insignificant variation in patterns across duty cycle.

Future work would involve investigating the normal forms of the symmetry breaking bifurcation to understand the resonance in the stripes and dots pattern seen with $T=127$ and $th=0.4$ and the pattern forming bifurcation between $th=0.4$ and 0.6 .

References

1. Becker C, Elliott MA (2006) Flicker-induced color and form: interdependencies and relation to stimulation frequency and phase. Conscious Cogn 15: 175–196.
2. Rule, M., Stoffregen, M., and Ermentrout, B. (2011). A model for the origin and properties of flicker-induced geometric phosphenes. PLoS Comput. Biol. 7, e1002158.
3. Dayan P, Abbott LF (2001) Theoretical neuroscience. Computational Neuroscience. CambridgeMA: MIT Press, xvi+460 pp. Computational and mathematical modeling of neural systems.