Generation of and Role for Cortical Traveling Waves

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Introduction

Experimental evidence shows a relationship between cortical phase waves and faint visual stimulus detection. Oscillations in the cortex become spatial phase gradients, which travel in alpha and theta waves. Voltage fluctuations go through the cortex as waves, which, based on the wave phase, have regions of hyperpolarization and depolarization. Faint visual stimuli are sometimes, but not consistently, detected. A study on the detection of faint visual stimuli in marmosets reported that waves should be depolarized above a threshold just before the onset of the stimulus for the stimulus to be detected by the animal.

In this study, models are proposed to mathematically show the generation of waves and the effect of phase of an oscillation on the likelihood of stimulus detection. We used the experimentally-derived suggestion that varying frequencies give way to wave generation to inspire our model.

Model

The Wilson-Cowan firing rate model represents spatial distribution of excitatory and inhibitory neurons in the primary visual cortex.

Frequency Gradient for Wave Generation:
The Wilson-Cowan model generates waves in a cortical layer.

\[
\begin{align*}
\frac{du}{dt} & = -u(t) + f(u(t)) - au(t) - b_u(t) + d_u(t); \\
\frac{dv}{dt} & = -v(t) + f(v(t)) - av(t) - b_v(t) + d_v(t)
\end{align*}
\]

where \(u\) is an excitatory neuronal population, \(v\) is an inhibitory neuronal population, \(f(u)\) is a nonlinear gain function representing firing rate, \(a\) is the strength of projection from network \(a\) to \(b\), \(b\) is the coupling factor between the local circuit and the rest of the network, \(d_u\) is a threshold for firing. \(K_m(x)\) is a kernel describing the weights of excitatory and inhibitory connections and is defined as \(K_m(x) = \frac{1}{\pi} \exp(-x^2)\).

The drive force to create variation in frequencies is defined as \(de(x) = \frac{1}{\tau_m} (de_1 - de_0) + de_2\).

Single Oscillation for Faint Stimulus Detection:
We first model a single oscillation occurring in the cortical layer.

\[
\begin{align*}
\frac{du}{dt} & = -u(t) + f(u(t)) - au(t) - b_u(t) + d_u(t) + C \sin \omega t \\
\frac{dv}{dt} & = -v(t) + f(v(t)) - av(t) - b_v(t) + d_v(t)
\end{align*}
\]

We then introduce noise and a stimulus from a sensory cortical layer where \(b_{\text{stim}}\) is the strength of projection from network \(a\) to \(b\) in the stimulating layer, \(a_{\text{noise}}\) determine the amplitude of the noise.

\[
\begin{align*}
\frac{du}{dt} & = -u(t) + f(u(t)) - au(t) - b_u(t) + sttm + a_{\text{noise}} u(t); \\
\frac{dv}{dt} & = -v(t) + f(v(t)) - av(t) - b_v(t) + stvm + a_{\text{noise}} v(t)
\end{align*}
\]

Low pass filtered noise:

\[
\begin{align*}
de_0 & = \frac{u(t) + u(t - \tau)}{2}; \\
de_1 & = \frac{u(t) - u(t - \tau)}{2}; \\
de_2 & = \frac{u(t) + u(t - \tau)}{2}
\end{align*}
\]

We measure the response due to the stimulus where \(\text{stim}\) is the time of stimulus onset and \(\text{wid}\) is the width of the Heaviside function.

\[
\begin{align*}
\text{stim} & = \text{amp} H(t + \text{wid} - 1) H(t - 1) \\
\text{wid} & = \text{amp} H(t + \text{wid} - 1)
\end{align*}
\]

The stimulus is a step function at time \(t\) of width \(\text{wid}\) and height \(\text{amp}\) where \(H\) is a Heaviside function.

Using the Wilson-Cowan model to generate waves, we look at the cross correlation between neurons connected across frequency plateaus versus across frequency jumps. The time scale for this analysis was increased to show greater variance in phase gradients.

References


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